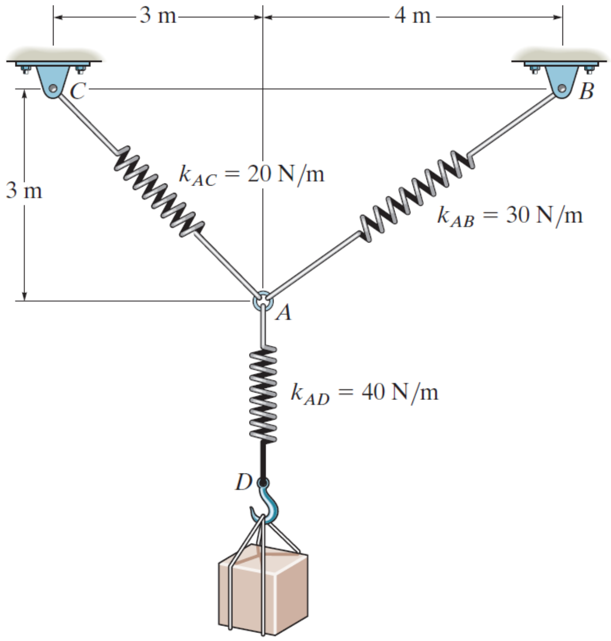
Problem 3-15

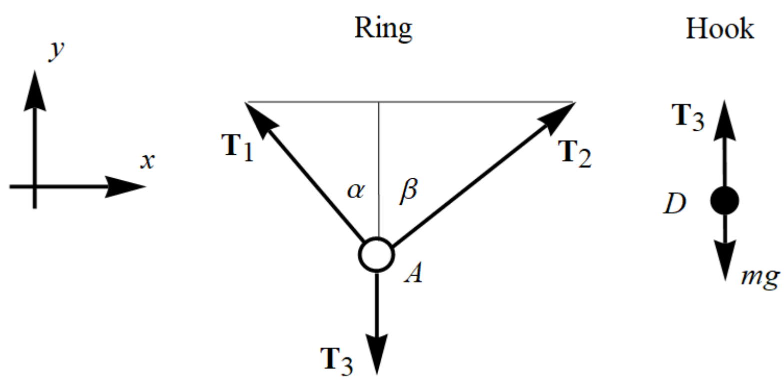
The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



Probs. 3–14/15

Solution

Draw one free-body diagram for the ring at A and one free-body diagram for the hook at D.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad -T_1 \sin\alpha + T_2 \sin\beta = 0 \qquad 0 = 0$$

$$\sum F_y = 0: \qquad T_1 \cos\alpha + T_2 \cos\beta - T_3 = 0 \qquad T_3 - mg = 0$$

Because the unstretched length of spring AB and k_{AB} are known, the magnitude of $\mathbf{T}_2$ can be found.

$$T_2 = k_{AB}\Delta x_{AB} = \left(30\ \frac{\text{N}}{\text{m}}\right)\left(\sqrt{3^2+4^2}\ \text{m} - 3\ \text{m}\right) = 60\ \text{N}$$

As a result, the system of equations for T_1 and T_3 becomes

$$-T_1 \sin\alpha + 60\sin\beta = 0 \tag{1}$$

$$T_1\cos\alpha + 60\cos\beta - T_3 = 0. \tag{2}$$

Solve equation (1) for T_1

$$T_1 = \frac{60\sin\beta}{\sin\alpha}$$

and substitute it into equation (2).

$$\left(\frac{60\sin\beta}{\sin\alpha}\right)\cos\alpha + 60\cos\beta - T_3 = 0$$

$$T_3 = \frac{60\sin\beta}{\sin\alpha}\cos\alpha + 60\cos\beta$$

$$T_3 = \frac{60\sin\beta\cos\alpha + 60\cos\beta\sin\alpha}{\sin\alpha}$$

$$T_3 = \frac{60\sin(\alpha+\beta)}{\sin\alpha}\ \text{N}$$

Use trigonometry to determine α and β.

$$\tan\alpha = \frac{3}{3} \quad \rightarrow \quad \alpha = \tan^{-1}(1) = 45^\circ$$

$$\tan\beta = \frac{4}{3} \quad \rightarrow \quad \beta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.1^\circ$$

Therefore, since $T_3 = mg$, the block mass is

$$m = \frac{T_3}{g} = \frac{60\sin(\alpha+\beta)}{(9.81)\sin\alpha}\ \text{kg} \approx 8.56\ \text{kg}.$$